Game Theoretic Rate Control for Mobile Devices

Dimitrios Tsamis
Electrical Engineering
Stanford University
dtsamis@stanford.edu

Tansu Alpcan
Deutsche Telekom Laboratories
Ernst-Reuter-Platz 7, 10587, Germany
tansu.alpcan@telekom.de

Nick Bambos
Electrical Engineering
Stanford University
bambos@stanford.edu

***Abstract*—Modern mobile devices offer increased connectivity to heterogeneous wireless networks such as WiFi and 3G. These networks typically exhibit high variability in their characteristics, which poses extra challenges to applications developed to run over them. To address this issue, we propose a game-theoretic rate control scheme for mobile devices. In this noncooperative game formulation of the system the users are assumed the to be selfish in terms of their resource (bandwidth) requests. The information requirements within the model are then simplified by exploiting the available bandwidth measurements provided by a tool called Zeus we developed for this purpose. The formulation is shown to be a potential game, admitting a unique Nash equilibrium. Furthermore, the resulting distributed gradient update algorithm is shown to converge globally. The validity of the model is verified through numerical analysis run on real data, collected by Zeus from different wireless networks.**

I. Introduction

Modern mobile devices, such as smartphones, are increasingly becoming open platforms which allow third parties to develop applications on them. They pose an attractive computing environment, in two main ways: first, users have shown that they are willing to carry them all all the time, i.e. user acceptance. Second, they have uninterrupted (Internet) connectivity either through the network carrier (UMTS, EDGE) or through WiFi access points. On the other hand, wireless networks, by their nature, are highly varying in their capacity and delay. Application developers can either take a best-effort approach or take measures to mitigate the effects of such variability. One such measure is the network assisted rate-control scheme that we are investigate in this paper.

Tasks related to distributed rate control such as bandwidth estimation can be realized with the help of a network assistance server (NAS) located at the edge of the network in close proximity (low latency) to the mobile devices. The paradigm of *network assisted computing* can play a significant role in next generation wireless computing as a vital architectural component. NAS supports mobile devices in heavy computational tasks and helps their battery preservation. Furthermore, it can provide assistance in optimization and network control tasks such as the ones described in this work and in general a range of capabilities such as content caching at network edges.

In this paper, we extend the distributed and network assisted rate control studies of [1] and [2]. Specifically, we consider a setting with users who are selfish in terms of their resource (bandwidth) requests. This is in contrast to the collaborative approaches in [1] and [2] where users respectfully back-off in order to prevent network congestion and allow new users to obtain a fair share of bandwidth. Consequently, we propose here a game-theoretic rate control scheme and utilize noncooperative game theory to study its properties. The scheme is shown to be a potential game, admitting a unique Nash equilibrium. Furthermore, the resulting distributed gradient update algorithm is shown to converge globally.

The information requirements of the game-theoretic rate control scheme are simplified by relying on observations of individual mobile devices. This can be achieved using the the available bandwidth measurements provided by a tool called Zeus we developed for this purpose. Such realistic measurements also provide the basis of our numerical analysis demonstrating the validity of the proposed scheme.

The main contributions of this work include:

- Resource allocation for mobile devices in a noncooperative setting with selfish users extending our earlier work [1], [2].
- A distributed rate control scheme based on noncooperative game theory and its analysis.
- Simplification of information dissemination within the game using a measurement-based approach.
- Numerical analysis demonstrating the theoretical results based on the realistic measurement data collected.

Notice that our work could also be applied to laptops and netbooks, as in their majority they can connect to 802.11 networks, as well as UMTS and EDGE in some cases. However, it should be noted that we specifically target smartphones with their limited resources within the context of network assisted computing. Laptops and netbooks are generally much more powerful and they could benefit from more complex models that can run on them.

The organization of the paper is as follows. In Section II we briefly discuss the rate control problem and previous work on it. In Section III we introduce and develop our specific network and game model and in Section IV we present a bandwidth measurement tool we have developed called Zeus. The measurements help simplify the game model and the stability of the simplified model is analyzed in Section V. In Section VI we test the performance of our model through simulations on data collected with Zeus. Finally, in Section VII we present the main results related to our model and discuss future work.

II. Game Theoretic Rate Control

Rate control refers to the problem of sharing the available bandwidth of a network among competing users and flows. The objectives may differ depending on the application: for example, in multimedia streaming applications it is generally desirable to avoid large fluctuations of the rate, whereas in bulk data transfers the goal is to achieve the highest possible rate. In general, a higher achievable rate offers a higher utility to the user, but when the aggregate rate exceeds the available bandwidth there are costs associated with the dropped packets.

In developing rate control mechanisms, game theory provides a natural framework. Users on the network can be modeled as players in a rate control game where they choose their strategies or in this case flow rates. A user's demand or utility for bandwidth is captured in a utility function. To compensate for this, one can devise a *pricing* function, proportional to the bandwidth usage of a user, as a disincentive to him to have excessive demand for bandwidth. This way, the network resources are preserved, and an incentive is provided for the user to implement end-to-end congestion control [3]. In cooperative games, groups of users may enforce contracts through third parties which results in centralized systems. In contrast, in non-cooperative games users may choose to cooperate, but any cooperation is self-enforcing. Since players are selfish in terms of their demands for network resources, and usually have no specific information on other users' strategies, i.e they play against the crowd, the choice of non-cooperative games is justified. A useful solution concept in such a noncooperative rate control game is that of Nash equilibrium [4] where each player minimizes his/her own cost (or maximize payoff) given all other players' strategies.

Before presenting the specific game theoretic framework developed in this paper, it is enlightening to review the conventional approaches. In his highly cited paper, Kelly [5] proposed a distributed optimization framework for rate control and since then there have been many extensions on it (see [6] for a survey). The most relevant ones refer to wireless networks (eg [7], [8]), where there are random fluctuations in the channel capacity. Another extension that introduces delivery constraints [9] was combined with Zeus and our channel model (work to appear in future publication). There is by now also a rich literature on game theoretic analysis of flow control problems utilizing both cooperative [10] and noncooperative [11]–[15] frameworks.

III. Network and Game Model

We consider a network model where multiple mobile devices $\mathcal{N} = \{1, 2, \ldots, i, \ldots, N\}$ connect to a single access point (AP). The problem of using multiple APs and choosing the best is a future direction of the research. In the single AP setting all devices effectively share a single (wireless) channel i.e. they compete for the same set of resources. In reality, due to fading phenomena the wireless channel appears to have different characteristics depending on the location of the user. This poses a very interesting observability problem which is covered in part in the following sections.

Let r_i be the current transmission rate of a user $i \in \mathcal{N}$ and $r := [r_1, \ldots, r_N]$ be the corresponding rate vector. The following analysis aims to define a rule to choose the rates r_i by solving a rate control game. Toward this end, we first define the action space of the game and then user cost functions. The action space of the users is defined as $X := \{r \in \mathbb{R}^N : r_i \geq 0 \ \forall i, \text{ and } \sum_i r_i \leq C\}$ for a given vector of rates r and channel capacity C. Each user i is associated with a cost function J_i on this action space. Naturally, the cost functions of each device necessarily include the current rates r as well. A general form for the cost function $J_i(r_i, r_{-i})$ of user i is:

$$J_i(r_i, r_{-i}) = g(r_i) + h(C, r) - U_i(r_i)$$

The last term, the user utility, measures the benefit the user enjoys by transmitting at r_i. It is a strictly increasing and concave function and in this paper it will be chosen to be logarithmic. The first term represents the usage-based cost of rate r_i, which includes the power consumed by the device to transmit at this rate, as well as the cost of any pricing policy enforced by the network. We adopt in this paper a linear function for g with a positive coefficient for simplicity. For the second term, h, notice that $\sum_i r_i$ is the aggregate bandwidth used by all devices. If it exceeds the channel capacity C, then packets will be dropped and hence such behavior should be penalized. This behavior can be prevented by choosing a penalty (barrier) function of the form:

$$h(C,r) = \frac{1}{C - \sum_j r_j}, \qquad \sum_j r_j \leq C$$

Thus we can write the cost function for each user (device) as:

$$J_i(r_i, r_{-i}) = \alpha_i r_i + \frac{1}{C - \sum_j r_j} - \beta_i \log(r_i) \quad (1)$$

where α_i and β_i are positive fixed and user-specific parameters (constants) respectively. Then, given the rate vector r, each user i minimizes its own cost J_i by choosing an appropriate rate r_i.

IV. Measurement Tool

As noted in the introduction, modern smartphones and PDAs can connect to many types of wireless networks, the main ones being WiFi (802.11g), 2nd Generation (2G, GPRS) and 3rd generation (3G, UMTS). These networks are heterogeneous, since the statistics of their capacities and delays are significantly different. It was decided that in order to get realistic results, it is necessary to measure available bandwidth and round-trip times on these networks and to collect real datasets.

To our knowledge, there are no vendor-supplied tools to perform these measurements and the scientific community has only sparse bibliography on the subject. There has been ample documentation on the issue of measuring available bandwidth (see for example [16]–[18]), but most of it focuses on personal computers and wired networks. Wireless networks have high variability, which dictates fast convergence times for the tools - otherwise their results are irrelevant. Mobile devices need extra attention, because of their limited computational power and reduced timer accuracy. To meet these challenges, we developed a new tool, called Zeus.

Most of the measurement tools require the cooperation of the two endpoint of a connection and it was chosen to maintain this design. In the wireless setting, one endpoint is the mobile device and the other one is the network assistance server (NAS), which could be the base station operated by the carrier. It is assumed that the NAS has enough processing power and accurate timers. These are very light assumptions which are easily met by many computing systems. Zeus takes advantage of this by having the NAS perform the heavy computations and send back the results. In addition, Zeus avoids performing network operations on the mobile device that require high-precision timers and instead relies on the timers of the NAS.

Zeus runs in two distinct phases, both of which are based on existing tools, which were combined to get the desired results. In the first phase, the effective capacity of the network is measured, as in the WBest tool [19]. In the second phase, the available bandwidth is derived from the effective capacity, as in the Spruce tool [20], which assumes that the capacity is known. Both series only require a series of 20 packet pairs. Thus, in total 40 packet pairs are sent, which makes Zeus non-intrusive and achieves convergence times of around one second.

Given a series of measurements of available bandwidth and round trip times, a model was build to make short-term predictions for these quantities. The model uses intelligent quantization of the states to remove the complexity of the learning phase and then a Markov Chain is built on the compressed state space. More details on this work can be found in [2]. Although not directly employed in the current paper, the short-term predictions could potentially improve the performance of the game model, as will be described in the next section.

Zeus is written in Python and can run on all Symbian S60 devices using the PyS60 package, a port of Python to said devices. By avoiding to program Zeus in a hardware specific manner, it is made easily portable to any other device that can run Python programs. The measurements were performed in the mobile services and security testbed (MoSST) located in Deutsche Telekom Laboratories. Multiple Nokia N80 devices were used, which run the Symbian S60 program and can connect to all of the above mentioned networks. The testbed also included an access point and a network assistance server.

V. Equilbrium and Stability Analysis

The term $C - \sum_i r_i$ in the cost function (1) complicates the game, by creating an environment where users are coupled (interact with each other) due to capacity constraints. They may either cooperate to find an optimal solution with respect to a chosen criterion or alternatively play selfishly a noncooperative game. In each case information requirements may vary due to specific implementation of dynamic algorithms computing these solutions. However, assuming that Zeus is available on the mobile device, its measurements can be exploited to handle this issue. Zeus measures available bandwidth, which is connected to the capacity through the following equation:

$$ABW = C - \sum_j r_j$$

Thus, we can rewrite the cost function 1 as:

$$J_i(r_i, r_{-i}) = \alpha_i r_i + \frac{1}{ABW} - \beta_i \log(r_i) \qquad (2)$$

Using this formulation each user can independently update her rate, ie the measurements collected by Zeus remove the necessity of sharing information among the users (although the measurements themselves are just another source of information). Both expressions of the cost functions are equivalent and in the rest of the paper we will use Equation (1) to analyze the game and Equation (2) in practical implementations of the scheme.

Note that this formulation includes only the current measurement and does not exploit the prediction model (or the measurement and predictions for the round-trip times). This extra information can be incorporated in the cost function J to let the device take early action to smooth future fluctuations. One way to achieve this is to modify the h function to $\sum_{t=1}^{5} h(t)$. This way (estimated) future constraints are taken into account, in addition to current. Another approach, similar to [9], is to turn the whole cost J to a sum over a finite time horizon. More analysis on these approaches will appear in future work.

The rate control game defined in the previous section is a potential game [21]. To show this, define the potential function ϕ:

$$\phi := \sum_i \alpha_i r_i + \frac{1}{C - \sum_i r_i} - \sum_i \beta_i \log(r_i).$$

Then, clearly,

$$\begin{aligned}\phi(r_i(1), r_{-i}) - &\phi(r_i(2), r_{-i}) \\ &= J_i(r_i(1), r_{-i}) - J_i(r_i(2), r_{-i})\end{aligned}$$

which makes this a potential game. Potential games have been extensively studied in the literature and many results are readily available. For example, in [22] it is shown that for repeated multi-player games which are also potential games, simple experimentation converges to a Nash equilibrium. More importantly, potential games always admit at least one Nash equilibrium [21]. Further analysis leads to the following result:

Theorem 5.1: The rate control game with the cost function defined in 1 admits a unique Nash equilibrium.

Proof: As mentioned above, potential games always admit a Nash equilibrium. Uniqueness of the NE is proven using Theorem 2.1 of [15] which states that the following are sufficient conditions to ensure existence and uniqueness of a NE in a N-player noncooperative game:

(i) The action space X of the game is defined as:

$$X := \{r \in \mathbb{R}^N : h_j(r) \leq 0, j = 1, 2, \cdots, m\},$$

where $h_j : \mathbb{R}^N \to \mathbb{R}$, $h_j(r)$ is convex in its arguments and the set X is bounded and has a nonempty interior. In addition, the derivative of at least of one of the constraints with respect to x_i, $\{dh_j(x)/dx_i, j = 1, 2, \cdots, m\}$ is nonzero for $i = 1, 2, \cdots, N, \forall r \in X$.

(ii) The cost function $J_i(r)$ is twice differentiable in all its arguments and it is strictly convex

(iii) The Jacobian $G(r)$ of $g(r)$ (the gradient of the cost function $J(r)$) is a positive definite matrix

The action space is $X := \{r \in \mathbb{R}^N : r_i \geq 0 \; \forall i$, and $\sum_i r_i \leq C\}$ and it is easy to see that condition (i) is satisfied. The cost function J_i is twice differentiable in all its arguments with:

$$\frac{\partial^2 J_i}{\partial r_i^2} = \frac{2}{\left(C - \sum_k r_k\right)^3} + \frac{\beta_i}{r_i^2}$$

$$\frac{\partial^2 J_i}{\partial r_i \partial r_j} = \frac{2}{\left(C - \sum_k r_k\right)^3}$$

J_i is also strictly convex since $\beta_i > 0$ and $\sum_k r_k \leq C$. Let $\theta_{ii} = \frac{\partial^2 J_i}{\partial r_i^2}$ and $\theta_{ij} = \frac{\partial^2 J_i}{\partial r_i \partial r_j}$. Then $G = [\theta_{ij}]$ and

$$G = \frac{2}{\left(C - \sum_k r_k\right)^3} \cdot \text{ones}(N) + \text{diag}(\frac{\beta_1}{r_1^2}, \cdots, \frac{\beta_N}{r_N^2})$$

where ones(N) is a $N \times N$ matrix where all entries are ones. It follows immediately that G is positive definite.

Hence, Proposition 2.2 of [15] holds and there can be at most one Nash equilibrium solution (which is the unique equilibrium in the case of potential games). ■

A gradient algorithm of type $\dot{r}_i = -\partial J_i / \partial r_i$ converges to this unique Nash equilibrium (assuming naively that rates are fixed). This follows immediately because one can show that the potential function ϕ acts as Lyapunov function at the same time. The assumption there is that updates occur on a faster time scale than capacity changes. This is of course an idealistic setting and we intend to use perturbation theory for further analysis. In the following section we evaluate the performance of our algorithm against a standard AIMD scheme using simulations.

VI. SIMULATIONS

We simulate the game theoretic rate control scheme using $N = 20$ users (devices) on three different types of wireless networks (WiFi, UMTS and GPRS), using real data collected by our measurement tool Zeus. In the first set of simulations all users are presented with the same set of information and we are interested in how the aggregate bandwidth used by all devices behaves with regards to the total capacity of the network. We also tested AIMD in the same setting to illustrate how they compare to each other. We then investigated a more realistic scenario where different users get different values of available bandwidth.

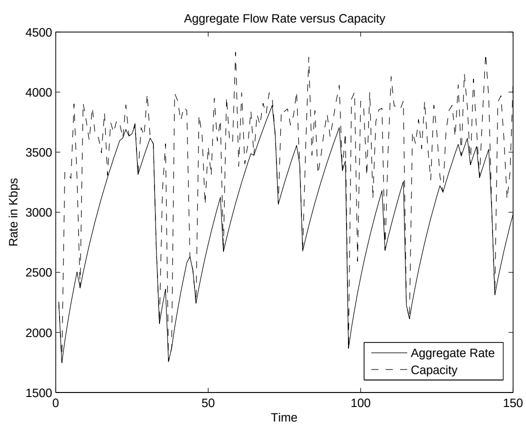

Fig. 1. Aggregate rate vs capacity for a WiFi network under game theoretic rate control

Figures 1, 2 and 3 present how game theoretic rate control performs on the three networks when all users observe the same available bandwidth. The aggregate rate as a percentage of the capacity is 85%, 98% and 89% respectively. Notice that the second network, UMTS, has much narrower band of measurements, most of them being between 370 and 400 kbps. This probably explains the exceptional performance of our algorithm in this network.

In comparison, figures 4, 5 and 6 show the results of AIMD for the same setting as before. In the AIMD scenario the rate of each increases by 10, 1 and .5 kbps

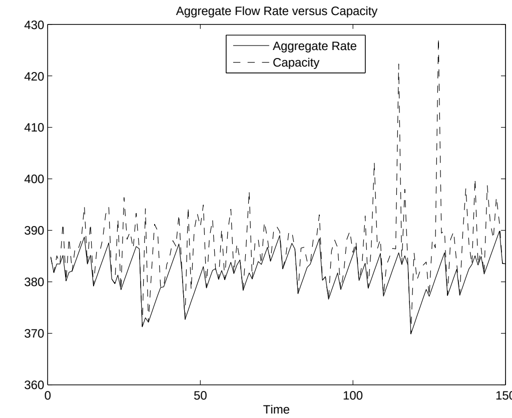

Fig. 2. Aggregate rate vs capacity for a UMTS network under game theoretic rate control

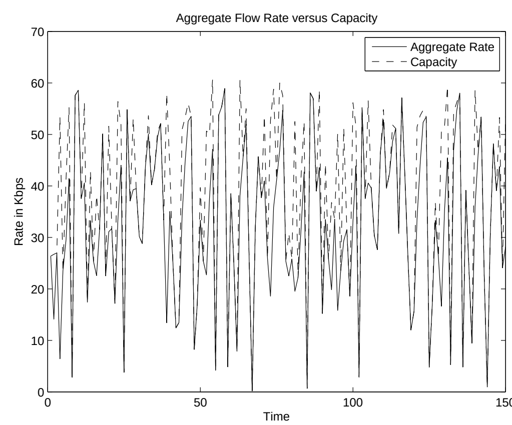

Fig. 3. Aggregate rate vs capacity for a GPRS network under game theoretic rate control

respectively for each network every time transmission is successful and decreases by 25% every time capacity is surpassed. This time the aggregate rate over capacity is 79%, 88% and 65% respectively.

We also introduced Gaussian noise $\mathcal{N}(0, 100)$ to the bandwidth measurements, presenting to each user a different view of the network. This is a more realistic scenario, which escapes the assumptions of our algorithm. However, in figure 7 it can be seen that game theoretic rate control continues to perform at almost the same level and the aggregate rate is now 78% of the capacity.

VII. CONCLUSION AND FUTURE WORK

The rate control problem for mobile devices has been formulated as a game and analyzed. The corresponding game is non-cooperative and thus each user decides

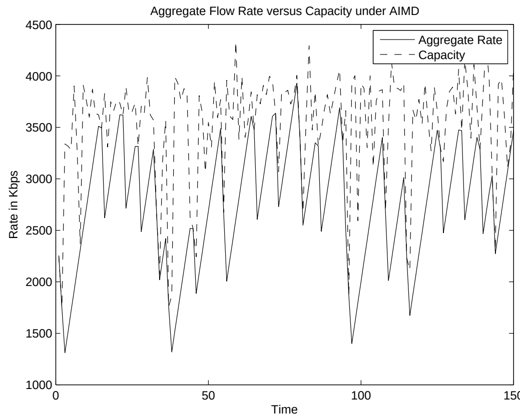

Fig. 4. Aggregate rate vs capacity for a WiFi network under AIMD

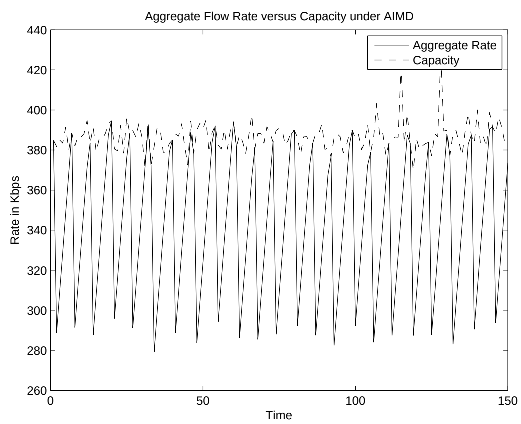

Fig. 5. Aggregate rate vs capacity for a UMTS network under AIMD

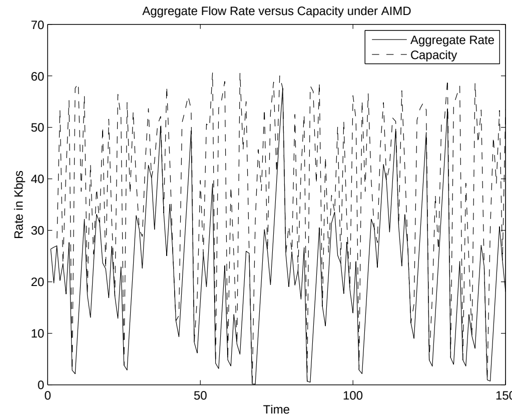

Fig. 6. Aggregate rate vs capacity for a GPRS network under AIMD

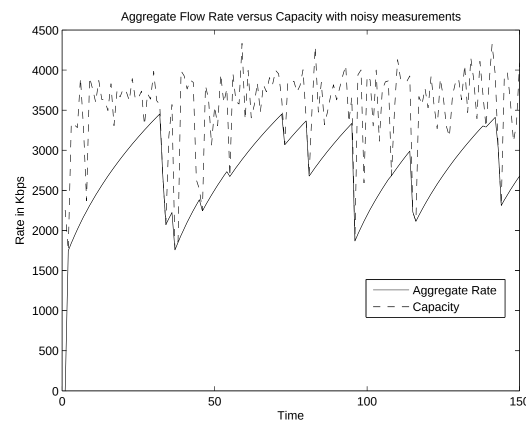

Fig. 7. Aggregate rate vs capacity for a WiFi network with noisy measurements under game theoretic rate control

independently in a decentralized fashion. A measurement tool, Zeus, allows the game players to be "decoupled" by estimating key game parameters. Through simulations it is shown that this approach performs consistently better than the standard AIMD (additive increase multiplicative decrease) strategy in both ideal and realistic scenarios.

Although not thoroughly analyzed in this paper, there are two time scales in this framework, since capacity in wireless networks is fluctuating over time. We plan to address this issue in future work using perturbation theory. Another future direction involves taking short-term bandwidth predictions (which we developed in another work) into account by including them in the cost function.

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