A Stackelberg Game for Power Control and Channel Allocation in Cognitive Radio Networks

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ABSTRACT

The ongoing growth in wireless communication continues to increase demand on the frequency spectrum. The current rigid frequency band allocation policy leads to a significant under-utilization of this scarce resource. However, recent policy changes by the Federal Communications Commission (FCC) and research directions suggested by the Defense Advanced Research Projects Agency (DARPA) have been focusing on wireless devices that can adaptively and intelligently adjust their transmission characteristics, which are known as *cognitive radios*. This paper suggests a game theoretical approach that allows master-slave cognitive radio pairs to update their transmission powers and frequencies simultaneously. This is shown to lead to an exact potential game, for which it is known that a particular update scheme converges to a Nash Equilibrium (NE). Next, a Stackelberg game model is presented for frequency bands where a licensed user has priority over opportunistic cognitive radios. We suggest a modification to the exact potential game discussed earlier that would allow a Stackelberg leader to charge a virtual price for communicating over a licensed channel. We investigate virtual price update algorithms for the leader and prove the convergence of a specific algorithm. Simulations performed in Matlab verify our convergence results and demonstrate the performance gains over alternative algorithms.

1. INTRODUCTION

As wireless communication devices become more pervasive, the demand for the frequency spectrum that serves as the underlying medium grows. Traditionally, the problem of allocating the resource of the frequency spectrum has been handled by granting organizations and companies licenses to broadcast at certain frequencies. This rigid approach leads to significant under-utilization of this scarce resource, as can
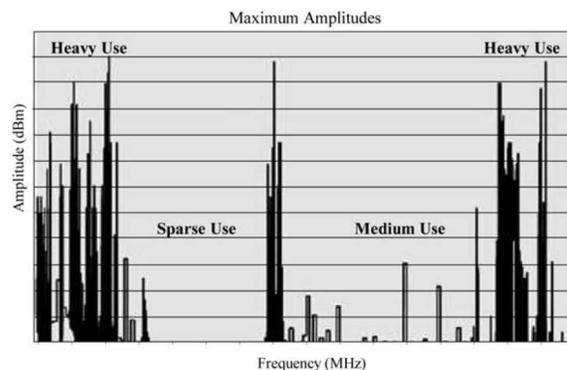
Figure 1: Much of the frequency spectrum is underutilized, even though most of it has been assigned [9].

be seen in Fig. 1. Moreover, frequency utilization varies significantly with time and location [2].

A *cognitive radio* is a wireless communication device that is aware of its capabilities, environment, and intended use, and can also learn new waveforms, models, or operational scenarios [15]. For example, cognitive radios may operate opportunistically to make use of unused spectrum or compete with other cognitive radios for a shared frequency band.

Recently, the federal government of the United States has changed its approach to managing the frequency spectrum. This paradigm shift is explained in [1], where the Federal Communications Commission (FCC) describes how cognitive radios can lead to more dynamic and efficient use of the frequency spectrum and proposes rule changes and certification tests for such radios. Moreover, the Defense Advanced Research Projects Agency (DARPA) has initiated "The NeXt Generation (XG) Program" in their Strategic Technologies Office to study how to "dynamically redistribute allocated spectrum" for defense purposes by making use of cognitive radio-like devices [9]. Cognitive radios are poised to alter the wireless communication landscape.

Cognitive radios dynamically interact such that the desirability of outcomes depends not only on their own actions but also on those of other cognitive radios. Therefore, cognitive radio system design is a natural context in which to apply game theory. This paper applies the Stackelberg game concept and virtual pricing mechanisms to cognitive radio networks.

1.1 Previous Work

An excellent survey of recent research on the topic of cognitive radios can be found in [2]. This paper discusses several varied aspects of cognitive radios, such as sensing of the environment and spectrum management. It also lays out some areas for future research. Two texts, [11] and [14], provide a more complete technical background to cognitive radio devices.

Other research investigates applications of game theory to cognitive radio networks. Neel *et. al.* make use of game theory tools and concepts such as potential games and S-modularity while studying cognitive radio networks in [15, 16]. Nie and Comaniciou apply potential games and Φ-no-regret learning to networks of cognitive radios in [18]. The research presented here is in many ways an extension to [18].

Game theory and pricing mechanisms have found fruitful application to a variety of important problems in computer and wireless networks. In [10], the Altmans apply S-modular games to the problem of power control in wireless networks. Game theory is also applied to power control in CDMA wireless networks in [4] and [5]. In [3], game theory is applied to multicell wireless data networks. The case when resources are divisible is considered in [13]. In [6], flow control is studied using game theoretical tools. Finally, pricing strategies for a leader or administrator are studied in [8] and [12].

1.2 Summary of Contributions

This paper contains two main contributions. The first one is to develop an algorithm that allows for transmission power and transmission frequencies to be chosen simultaneously by cognitive radios competing to communicate over a frequency spectrum. This algorithm is developed by constructing utility functions for each radio such that we have an *exact potential game (EPG)*. We will define EPG in Section 3; for now it will suffice to say that EPGs exhibit attractive convergence and stability properties. Most of the literature on cognitive radio networks has only considered the transmission power and transmission frequency problems separately.

The second contribution of this paper is to apply the Stackelberg game concept to cognitive radio networks. In a Stackelberg game, there exists a leader who declares his strategy before other players and then enforces it [7]. Cognitive radios may interact opportunistically on frequency bands that are owned by licensees whose communication needs take priority over those of the cognitive radios. Therefore, we assign the role of Stackelberg leader to the licensee of a frequency band and assume that cognitive radios are followers. Stackelberg games are a natural fit for this scenario, but we are unaware of any other attempts to apply Stackelberg game theory to cognitive radio networks.

The remainder of the paper is organized as follows: In Section 2, we introduce our model and the problem we consider. We develop an EPG for simultaneous power and channel control in cognitive radio networks in Section 3. In Section 4 we introduce the Stackelberg game that models the interaction between a licensee of a portion of frequency spectrum and cognitive radios that opportunistically make use of this spectrum. Section 5 contains simulations of the game theoretical models we have introduced and demonstrates performance gains that are achieved by utilizing the update algorithms we propose. Finally, in Section 6 we summarize our results and propose several promising directions for future research.

2. SYSTEM MODEL

The model that we introduce is in many ways similar to that studied in [18]. We consider N master-slave radio pairs operating in an area of dimension $D \times D$. This problem formulation is quite general and can easily be modified to consider other cases, such as radios transmitting to various base stations.

Actions for a player i in the game (a master-slave pair) are a power level p_i in the set of power levels $P = (p^1, p^2, \ldots, p^m)$ and a channel c_i in the set of channels $C = (c^1, c^2, \ldots, c^l)$. These can be combined into a composite action (a pair) $s_i = (c_i, p_i) \in S_i$. Therefore, the entire action space is defined by $\mathbb{S} = \times S_i, i \in N$.

The signal to interference ratio (SIR) for a master-slave pair i is given by

$$\gamma_i := \frac{Lp_i h_{ii}}{\sum_{k=1, k \neq i}^{N} p_k h_{ki} f(c_k, c_i) + \sigma}, \qquad (1)$$

where γ_i specifies the SIR for the pair i, h_{ij} is the link gain from the transmitter in pair i to the receiver in pair j, σ is the ambient noise on each channel (assumed to be the same on all channels), and f is defined as

$$f(c_k, c_j) := \begin{cases} 1 & \text{if } c_k = c_j \\ 0 & \text{else} \end{cases}. \qquad (2)$$

When we study the Stackelberg game scenario in Section 4, we must consider also the noise generated by the leader's action pair when computing SIR values. Therefore, (1) is then modified to:

$$\gamma_i := \frac{p_i h_{ii}}{\sum_{k=1, k \neq i}^{N} p_k h_{ki} f(c_k, c_i) + p_l h_{li} f(c_l, c_i) + \sigma}, \qquad (3)$$

where p_l denotes the power level of the leader, and h_{li} is the link gain from the transmitter in the leader pair to the receiver in pair i. Also, note that the SIR for the leader, γ_l, will be as in (1), except that we will sum over all $k = 1$ to $k = N$ in the denominator.

Our objective is to find power and channel selections such that all cognitive radio pairs achieve as high of a utility value as possible, but also such that the leader receives a minimum required SIR level of γ^*. This objective will be defined and quantified precisely in Sections 3 and 4, with the choices also driven by the desire to prove the stability and convergence of the algorithms we obtain using game theory.

3. A POTENTIAL GAME FOR SIMULTANEOUS POWER AND CHANNEL UPDATES

In order to develop an algorithm that allows for optimization over action spaces that include power levels as well as transmission frequencies, we specify a carefully constructed utility function for each player i. The utility for each player is the sum of three terms, which we will introduce individually. The first term captures the impact that other players have on the interference sensed by the receiver in the pair i:

$$T_i^1(s_i, s_{-i}) := - \sum_{j \neq i, j=1}^{N} p_j h_{ji} f(c_j, c_i). \qquad (4)$$

Here s_i denotes a particular action pair chosen by player i and s_{-i} denotes the action pairs for all other players. Note

that this term is part of the denominator of the SIR equation (1) multiplied by -1, so maximizing it increases the SIR for player i.

The next term in the utility for radio pair i is

$$T_i^2(s_i, s_{-i}) := -\sum_{j \neq i, j=1}^{N} p_i h_{ij} f(c_i, c_j). \tag{5}$$

This term is very similar in structure to $T_i^1(s)$, except that it captures the impact of a potential action for player i on the interference observed by all other players. Therefore, by including the negative of this quantity in the utility for player i we cause players to cooperate.

The third and final term in the utility for player i is given by

$$\begin{aligned} T_i^3(s_i, s_{-i}) := & \alpha \log(1 + p_i h_{ii}) + \beta/p_i \\ & - p_l h_{li} f(c_l, c_i) - p_i h_{il} f(c_i, c_l). \end{aligned} \tag{6}$$

This term depends only on the action selected by player i and provides an incentive for individual players to increase their power levels. Note that utility increases logarithmically with increased power levels. By choosing this form we assume that players receive diminishing marginal utility with increasing bandwidth, a standard assumption in network theory. We weight the part of this term that incentivizes larger power levels by α so that we can give it more or less importance than other parts of the utility function.

The β/p_i term in this equation takes into account the utility associated with longer battery life. Battery life is proportional to the inverse of the transmission power, and the β parameter allows us to give this aspect of the utility more or less weight than other aspects. The relative choices of α and β capture the tradeoff between an incentive to increase the individual power level so as to have a higher level of SIR and the incentive to conserve power for future usage.

Finally, the last two terms in T_i^3 capture the impact that a player's action pair has on the leader, and vice versa.

We thereby arrive at the utility function for a radio pair i, which maps $\mathbb{S}$ into $\mathbb{R}$ for each player:

$$\begin{aligned} U_i(s_i, s_{-i}) := & T_i^1(s) + T_i^2(s) + T_i^3(s) \\ & \forall i = 1, 2, \ldots, N. \end{aligned} \tag{7}$$

This utility function is a modified version of that used in [18]. It quantifies the weighted tradeoff between cooperative behavior (as captured by the first two terms) and selfish attitude (as captured by at least part of the third term).

While we will demonstrate that this utility function has several desirable characteristics, we must note that in order to maximize this utility at any given time, a radio pair must have access to a significant amount of information. This information is communicated over a common communication channel and stored by the receiver in each radio pair in a *Channel Status Table*, as described in Section III.A of [18]. See Section 6 for a discussion on how to reduce this communication overhead.

We will demonstrate that the game with utility functions given by (7) is an EPG. An EPG is a game for which there exists a *potential function*, $V(s)$, which is a function from $\mathbb{S}$ to $\mathbb{R}$ with the property that

$$U_i(s_i, s_{-i}) - U_i(s_i', s_{-i}) = V(s_i, s_{-i}) - V(s_i', s_{-i}) \tag{8}$$

for all $i \in N$ and for all $s_i, s_i' \in S_i$. This means that when one player at a time changes its action (or strategy in a broader context), the change in the potential of the game is the same as the change in the utility of the acting player.

We now note that the following expression for $V(S)$ is an exact potential function for this game:

$$\begin{aligned} V(s_i, s_{-i}) := & \sum_{k=1}^{N} \left(\frac{1}{2} T_k^1(s) + \frac{1}{2} T_k^2(s) + T_k^3(s) \right) \\ & \forall i = 1, 2, \ldots, N. \end{aligned} \tag{9}$$

PROPOSITION 3.1. *The game defined by utility functions (7) and the potential function (9) is an exact potential game.*

For a proof of this proposition see the Appendix.

REMARK 3.2. *Since the game under consideration is an EPG defined on a finite set of actions, if*

a) *only one player acts at each time step, and*

b) *the acting player maximizes its utility, given the most recent actions of the other players*

then the process leads to an update algorithm which will converge to a Nash equilibrium (NE) [15], regardless of

i) *the order of play, and*

ii) *the initial condition of the game.*

Note that a set of actions $s^* = [s_1, s_2, \ldots, s_N] \in \mathbb{S}$ constitutes a NE when no player has an incentive to change its action pair unilaterally:

$$U_i(s^*) \geq U_i(s_i, s_{-i}^*), \forall i \in N, s_i \in S_i. \tag{10}$$

Remark 3.2 says that at least one (pure-strategy) NE exists for any EPG defined on a finite set of strategies. In general strategy spaces, however, some additional conditions are needed for the potential function to have a global maximum (such as upper semi-continuity on a compact set of strategies), which would then correspond to a Nash equilibrium.

While the update algorithm specified by Remark 3.2 does converge to a NE regardless of the order of play and the initial condition of the game, the particular NE that it converges to may depend on these factors.

When the players in a game share an objective function, a NE is known as a player-by-player maximum. In an EPG, the players act as though they are maximizing the potential function, but taking turns one at a time. Therefore, the update algorithm in Remark 3.2 converges to a player-by-player maximum of the potential function, and not necessarily to its global maximum.

It is restrictive to assume that only one player acts at each time step. Less coordination would be required if players could update their strategies in parallel or randomly. To avoid this coordination, players may choose to act in each stage with probability $1/N$. This approach is taken in [18], but the authors offer no proof of convergence when this method of choosing who will act is applied. We will use this approach, but we also suggest that this method converges when players act with any probability less than one. This result will be verified with simulations in Section 5.

4. STACKELBERG GAMES FOR COGNITIVE RADIO NETWORKS

4.1 Traditional Cognitive Radio Stackelberg Game

In many cognitive radio frameworks, such as XG networks, opportunistic cognitive radios operating on licensed frequency bands must evacuate the frequency when the owner starts transmitting [2, 11]. Clearly, this scenario involves a leader declaring a strategy that the other players in a game must obey, and so it could be modeled as a Stackelberg game.

In this traditional Stackelberg game, the cognitive radio pairs play the game specified in Section 3 but with one additional rule: if they sense the leader using a particular channel, then they are not allowed to use this channel. This game structure guarantees good performance for the leader, but may also be wasteful if the leader could share the licensed channel with other radios and still achieve a minimum required SIR.

4.2 Proposed Cognitive Radio Stackelberg Game

The current Stackelberg game for licensed frequency bands is potentially inefficient and leaves room for improvement. A different Stackelberg game can achieve quality of service (QoS) guarantees for the owner of a frequency band while allowing opportunistic cognitive radios to use the band as well. We assume that the leader has a minimum required SIR, γ_l^*, corresponding to a required bit error rate. A Stackelberg game algorithm is designed here to ensure this QoS for the leader at all times once we achieve convergence.

In this algorithm the leader can charge a virtual price for using the licensed frequency band. Call the licensed band $q \in C$, and define the virtual price as $W^q(c_i)$ for a given player i. If $c_i \neq q$, then the virtual price is zero. If $c_i = q$, then a nonnegative virtual price $W^q(c_i)$ is incurred by player i. We constrain $W^q(c_i)$ to be greater than or equal to zero, as a negative virtual price would harmfully distort the frequency channel allocation.

The introduction of this virtual price leads to a slightly different Stackelberg game than that in subsection 4.1. We change the utility function for a player to be

$$\tilde{U}_i(s_i, s_{-i}) := U_i(s_i, s_{-i}) - W^q(c_i)$$
$$\forall i = 1, 2, \ldots, N. \tag{11}$$

This modification to the utility function also requires the definition of a new potential function. If we set the potential to be

$$\tilde{V}(s_i, s_{-i}) := V(s_i, s_{-i}) - \sum_{k=1}^{N} W^q(c_k)$$
$$\forall i = 1, 2, \ldots, N. \tag{12}$$

then one can easily see that this game is also an exact potential game (for each fixed W^q), with all of the desirable characteristics thereof (see Section 3).

PROPOSITION 4.1. *The game defined by utility functions (11) and the potential function (12) is an exact potential game.*

Next we need to specify how the Stackelberg leader will update the virtual price. The leader's objective is to ensure that $\gamma_l > \gamma_l^*$ at any given time. However, the virtual price should not be set excessively high so as to prevent other users from accessing the channel, if possible. Therefore, we propose the following virtual price adjustment scheme (defined of course for $c_i = q$):

$$W_{k+1}^q = \begin{cases} 0 & \text{if } \gamma_l^* < \gamma_l < \gamma_l^* + \epsilon \\ [W_k^q + \lambda(\gamma_l^* - \gamma_l)]^+ & \text{else} \end{cases}, \tag{13}$$

where λ is a parameter, ϵ is a small positive constant, γ_l^* is the desired SIR and γ_l is the measured SIR at iteration k. Also, $[x]^+$ denotes 0 if $x < 0$, otherwise it is equal to x. We will refer to this virtual price update algorithm as the *bidirectional price update*, as it moves the virtual in both directions.

Note that each iteration for the leader's update should contain enough time steps for the system to converge to an equilibrium. In most cases, the algorithm is observed to converge after $3N$ time steps, so a new iteration of k is chosen to occur after every $3N$ time steps. In general terms, the price updates take place on a slower time scale than the iterative computation of the NE by cognitive radio master-slave pairs.

The convergence of this update algorithm has yet to be proven. In the absence of such a proof we utilize a simpler update algorithm for which convergence is shown. This algorithm starts at $W_0^q = 0$ and updates according to

$$W_{k+1}^q = \begin{cases} W_k^q + \Delta & \text{if } \gamma_l < \gamma_l^* \\ W_k^q & \text{if } \gamma_l \geq \gamma_l^* \end{cases}, \tag{14}$$

where Δ is a parameter that the leader can choose. A higher Δ leads to faster convergence but a lower Δ is more likely to allow other cognitive radios to use the licensed channel. We refer to this update algorithm as the *unidirectional update algorithm*.

To prove the convergence of (14), we must assume that the required γ_l^* is not set too high. More precisely, we define the upper bound on γ_l^* to be (which could be a loose bound):

$$\gamma_l^* < \frac{L p_l h_l}{\sigma}. \tag{15}$$

We then note that according to Proposition 4.1, this game converges to a NE for any virtual price W^q that the leader sets. We will show that for a high enough W^q, an upper bound on γ_l is achieved. We capture this result in the proposition below.

PROPOSITION 4.2. *There exists a $\bar{W}^q$ such that for every $W^q > \bar{W}^q$, the game defined by the utility functions (11) converges to a NE where $\gamma_l > \gamma_l^*$.*

PROOF. Define $\tilde{U}_{min}$ as the lowest utility that can be achieved by any player in the game defined by utility functions (11) when the player is not transmitting on the leader's channel:

$$\tilde{U}_{min} = \min_{i, p_i}[-(N-1)p_{max} - (N-1)p_i$$
$$+ \alpha \log(1 + p_i) + \beta/p_i], \tag{16}$$

where p_{max} is the highest possible power value. We know this minimum exists because the number of players and player action sets are both finite.
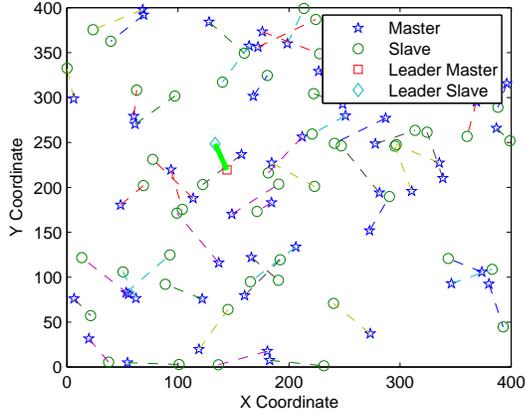
Figure 2: Sample placement of master-slave cognitive radio pairs for simulations.

Next, we set $\bar{W}^q = -\tilde{U}_{min}$. If this is the case, then no matter how low a player's utility is, there will never be an incentive to start transmitting on the leader's channel. Therefore, the leader would achieve an SIR of $\gamma_l = \frac{Lp_l h_l}{\sigma}$. We assumed in (15) that γ_l^* is less than this quantity, so $\gamma_l > \gamma_l^*$. $\square$

With this proposition, we know that eventually the update (14) will converge to a satisfactory equilibrium, because the virtual price W^q will grow incessantly until this is so.

This algorithm is limited in several ways, however. It is not guaranteed that the Stackelberg leader will end up setting the virtual price so as to allow other cognitive radios to use the licensed frequency. Moreover, the algorithm should be restarted if the network becomes significantly less congested, but this is not specified. The update algorithm suggested in (13) fixes these problems, but its convergence proof is yet out of reach.

In comparison with the traditional Stackelberg game framework, this proposed framework requires that the licensees of spectrum become more intelligent and complex. Thus, the leader would incur a cost when switching to this algorithm, which may prevent implementation in many contexts. However, this work also applies directly to cognitive radio networks where some radios have QoS guarantees on certain frequency bands.

5. SIMULATIONS

5.1 Simultaneous Power and Channel Updates

For the simulations we considered a network of $N = 50$ cognitive radio master-slave pairs placed randomly on a square with side length $D = 400$. The distance between a transmitting and receiving radio pair is assumed to be normally distributed with mean 30 and variance 15. A sample placement of the radios is shown in Fig. 2.

We use a link gain of $h_{ii} = (10/d_{ii})^2$, where d_{ii} is the distance from transmitter i to receiver i. If this distance is less than 10, we set the link gain to one.

The action spaces of the players consist of $l = 4$ channels and $m = 4$ power levels uniformly distributed between 250 and 1000. For σ we use 1, and the spreading gain is set to

$L = 128$. The α parameter is set to 50 and β is set to 10, so as to put a greater weight on improved communication than on battery life in the utility functions.

First we demonstrate the benefits of an algorithm that enables players to optimize their utility over transmission power as well as transmission channel. In each of these simulations the Stackelberg leader does not choose to transmit any data. We simulate our algorithm when only transmission power changes are allowed and all players operate on the same channel, when only channel changes are allowed and all players transmit at the maximum power level, and when both power level and channel changes are allowed. In each case the radios are placed at the same locations, and the same random initial conditions are assigned to the radios (power levels and channel selections).

The convergence of action updates in the game where players can change both power levels and channels is shown in Fig. 3. Note in Fig. 3 (a) how the players choose a variety of power levels, and how in Fig. 3 (b) the radios disperse so as to transmit on a variety of channels.
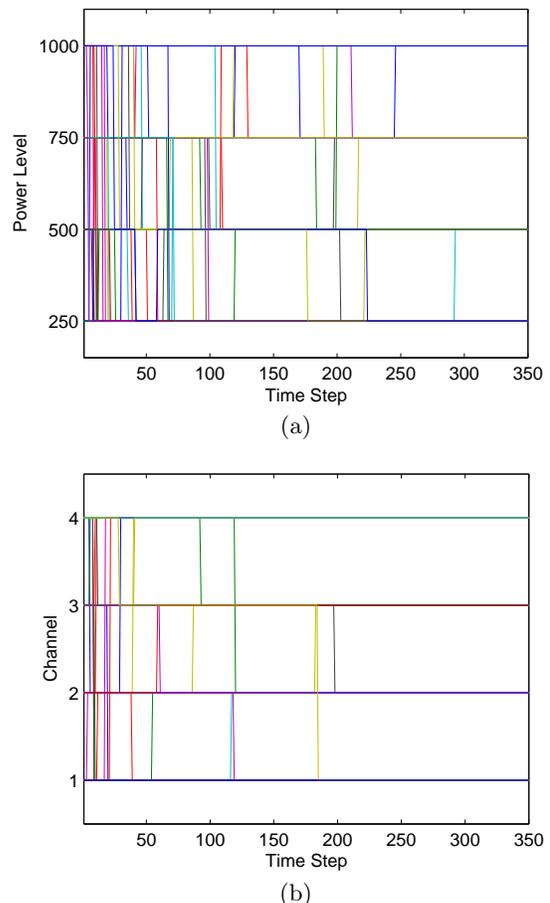

(a)

(b)

Figure 3: Convergence of cognitive radio pair (a) transmission level and (b) channel strategies.

We investigate the histograms of the SIR values for the cognitive radio pairs after the update algorithm has converged in each scenario. A plot of these histograms can be seen in Fig. 4.

Note that the final distribution of SIR values when we al-
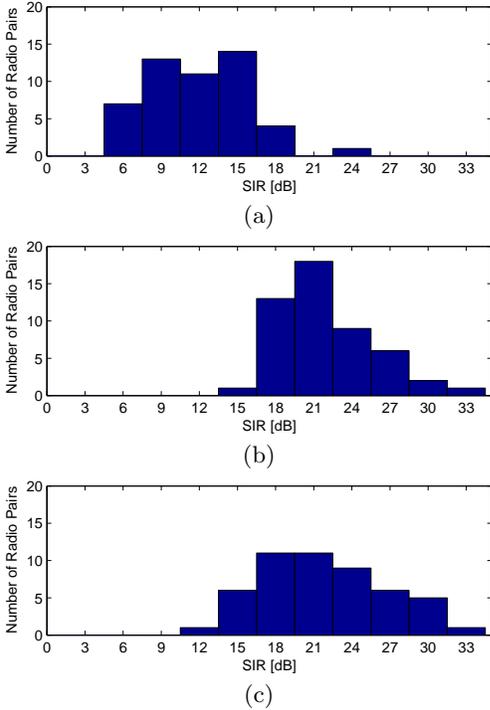
Figure 4: Histograms of radio SIR values when player strategies are (a) power levels only, (b), channels only, and (c) both power levels and channels.

Table 1: Total Utility and Battery Life Results Various Strategy Spaces

Strategies	Total Utility	Normalized Battery Life
Power Only	-10 650	4.0
Channel Only	-1016	1.0
Power and Channel	1439	2.1

low only channels to be adjusted is slightly preferable to the distribution when power levels and channels can be adjusted.

The total sum of all player utilities after convergence is shown in Table 1. We use the total utility as a metric for evaluating the quality of the final system state as it reflects the total value of the state to all radio pairs. Moreover, the structure of the utility functions in this game are not completely selfish, so a high total utility will not occur simply because a few players have done very well while others have not. This table also shows the average battery life, normalized such that the average battery life when all radios transmit at the highest power level is 1.

In the relatively congested scenario under consideration, communication becomes very difficult when all players are required to communicate on the same channel, even when power level updates are allowed. Therefore, the game that allows only power level updates performs poorly. The radio pairs end up turning their power levels as low as possible in this scenario in order to reduce interference, which does improve battery life. When we allow players to change their

channel but not their transmission power, the final total utility of the game is significantly improved. However, the best performance is seen when players optimize over both power levels and transmission channels. In this case the total final utility is 242% higher and the battery life is 110% longer than when players can only adjust channels.

We also performed simulations to determine the upper bound on the probability with which a player acts in each time step for which the update algorithm still converges. Our simulation results indicate that *any* probability of acting less than 1 leads to an update algorithm that converges eventually. However, convergence seems to be fastest when the probability of acting is set to relatively low values, near $1/N$. For probabilities greater than $1/N$, we speculate that the expected convergence time increases monotonically as the probability of acting increases.

5.2 Stackelberg Games

To demonstrate the performance of the suggested Stackelberg game formulations, we simulated the same scenario as in subsection 5.1 and allowed players to adjust channels and power levels. Parameters specific to the virtual price update scheme were set to $\epsilon = 3$ dB, $\lambda = 2.5$, and $\Delta = 15$ after trial and error tuning. The required/targeted SIR level for the leader is set to $\gamma_l^* = 20$ dB.

5.2.1 Traditional Stackelberg Game

First we simulated the traditional Stackelberg game, described in subsection 4.1. After 120 time steps, the leader begins transmitting on channel $q = 4$, and the other radios on the network evacuate that frequency channel immediately. They disperse by maximizing the utilities defined in (7). The final total utility value for this game was 28.29.

5.2.2 Proposed Stackelberg Game with Unidirectional Price Update

Here we simulated the Stackelberg game defined by utilities (11) and the virtual price update algorithm (14). The leader begins transmitting at time step 120. The final total utility for the game is 1082, which is 38 times higher than that achieved by the traditional leader. Note that this final total utility calculation does not include the virtual prices, as these are only mechanisms to ensure QoS for the leader, not indicators of the actual utility of the cognitive radio pairs. The improvement can also be seen in Fig. 5, which shows the histograms of the final SIR values in each Stackelberg game.

Figure 6 shows the SIR for the leader and the corresponding virtual price W_q over time.

Note how the virtual price increases until $\gamma_l > 20 = \gamma_l^*$, at which point the virtual price stops increasing. With this algorithm, several other cognitive radio pairs still utilize the leader's frequency channel.

5.2.3 Proposed Stackelberg Game with Bidirectional Price Update

Here we simulated the Stackelberg game defined by utilities (11) and the virtual price update algorithm (13). The final total utility in this case is 927.5, which is actually slightly lower than the final total utility when the unidirectional virtual price update is used, but still significantly higher than when the traditional Stackelberg game is played.

We plot the leader's SIR and the virtual price W^q over
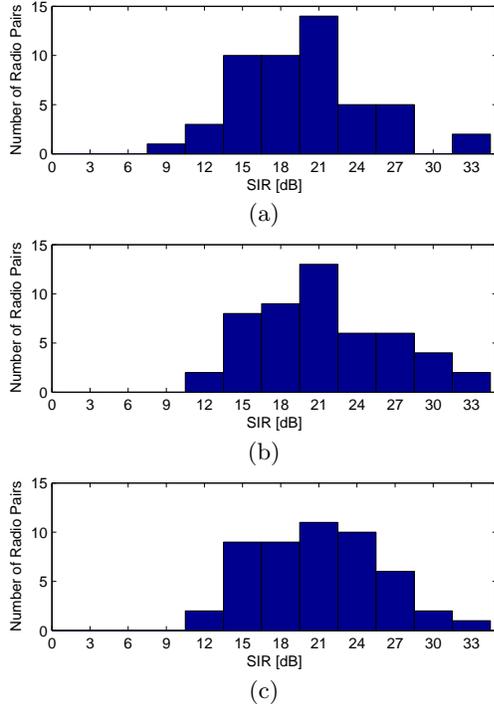
(a)

(b)

(c)

Figure 5: Histograms of radio pair SIR values when the Stackelberg leader (a) forces all radios to leave the licensed band, (b) uses a virtual price with the unidirectional update (14), and (c) uses a virtual price with the bidirectional update (13).
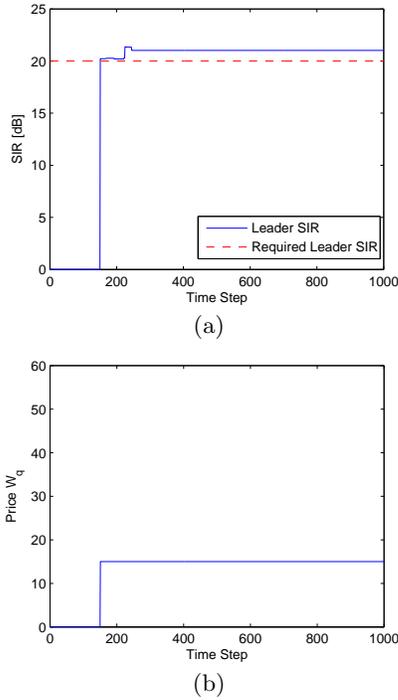

(a)

(b)

Figure 6: The leader's (a) SIR and (b) the virtual price W_q under the unidirectional virtual price update algorithm specified by (14).

time in Fig. 7. Note that here the algorithm converges nicely to a desirable solution, although some simulations of this virtual price update algorithm display chatter rather than true convergence. In this case six other radio pairs use the leader's frequency channel at the end of the simulation, as can be seen in Fig. 8.
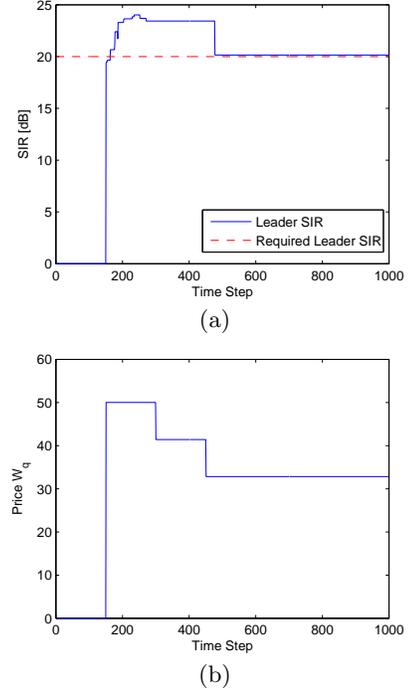

(a)

(b)

Figure 7: The leader's (a) SIR and (b) the virtual price W_q under the bidirectional virtual price update algorithm specified by (13).
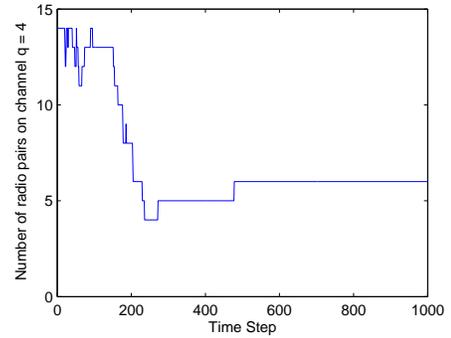

Figure 8: The number of cognitive radio pairs transmitting on channel $q = 4$ when the bidirectional virtual price update algorithm (13) is applied.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have introduced a decentralized update algorithm that allocates several frequency bands to cognitive radios by allowing them to maximize their utility by changing their transmission power and transmission channel simultaneously. We show convergence of this algorithm

by using well-known results for EPGs. While the algorithm is new in that it allows players to simultaneously update power levels and transmission channels, it also requires some amount of communication between the radios. Simulations of this algorithm verify its convergence properties and also demonstrate its superiority in terms of resulting total utility, SIR values, and average battery life.

Next we have suggested a Stackelberg game model for licensed frequency channels in which the licensee of the channel is the Stackelberg leader. After modeling the current paradigm for licensed frequency bands as a Stackelberg game, we have proposed a Stackelberg game in which the Stackelberg leader transmits a virtual price for using the licensed frequency band that impacts player utilities. Two price update algorithms for this virtual price have been proposed, with a proof of convergence included for one of them. Simulations have verified that total utility increases when this virtual pricing Stackelberg game is played rather than the traditional Stackelberg game.

The model developed and simulated here could be improved and expanded in many ways. For example, our speculation regarding the random selection of the acting player at each time step must be proven. An algorithm using less signaling overhead is preferable, such as the Φ-no-regret method used in [18]. Moreover, the convergence of the virtual price update algorithm (13) must be established. Of course there are numerous other possible update algorithms and even more possible Stackelberg game formulations for cognitive radio networks. The Stackelberg game concept fits very well in this context, and we expect that future research applying it to cognitive radios will be fruitful.

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APPENDIX

PROOF OF PROPOSITION 3.1

We first note that utility functions composed of the sum of $T_i^1(s)$ and $T_i^2(s)$ are identical to those used in [18]. For these utility functions the authors of [18] define a potential function that is equivalent to

$$V'(s) := \sum_{k=1}^{N} \left(\frac{1}{2} T_k^1(s) + \frac{1}{2} T_k^2(s) \right) \qquad (17)$$

and state that this is an exact potential function. This is proven in the journal version of their paper [17].

Next, we note that utility functions composed of only $T_i^3(s)$ make up a *self-motivated game*. A self-motivated game is one where a player's utility depends only on its own actions [16]. While the actions of the leader do affect the value of $T_i^3(s)$, the leader holds its actions constant while the opportunistic cognitive radios play this game. For a self-

motivated game, it is easy to show that a potential function defined as the sum of all the players' utilities works as an exact potential function. In this case, the exact potential function would be

$$V''(s) := \sum_{k=1}^{N} T_k^3(s). \tag{18}$$

Finally, consider a new game. Suppose we define the new utilities for each player such that they are the sum of scaled versions of the utilities for each player from other EPGs. Then, if we define a new function that is the sum of similarly scaled versions of the exact potential functions corresponding to each of these EPGs, then it is an exact potential function for the new game. This can easily be verified by using the associative and distributive properties of addition.

Therefore, if we assign player i a utility function (7) that is the sum of $T_i^1(s)$, $T_i^2(s)$, and $T_i^3(s)$ for all i, then the potential function (9) that is the sum of $V'(s)$ and $V''(s)$ is an exact potential function for this new set of utility functions. This proves that the game defined by utility functions (7) and the potential function (9) is an EPG. □